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NMR in Superfluid A-like Phase of ^3He Confined in Globally Deformed Aerogel in Tilted Magnetic Field.

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Abstract NMR spectra in superfluid A-like phases confined in axially deformed aerogel in presence of a magnetic field inclined with respect to deformation axis is considered. The characteristic features of dipole frequency shift in axially compressed and axially stretched cases are compared. In particular, it is shown that in axially stretched aerogel environment the stability region of coherently spin precessing mode is rather narrow due to the $U(1)$ LIM effect.

Keywords Superfluid ^3He · Aerogel

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1 Introduction

The NMR signature of spin-precessing modes of superfluid A-like phase immersed in a uniaxially deformed aerogel attracts vivid interest. An important information was collected from NMR spectra of the A-like phase in axially compressed [1,2] and axially stretched [3] cases. In this situation the orbital anisotropy axis \hat{l} of the order parameter of superfluid ^3He -A is pinned either along cylindrical axis (in axially compressed aerogel), or in the transverse plane (in axially stretched aerogel).

The main body of information on the peculiarities of coherent spin precession is contained in the transverse NMR frequency

$$\omega_{\perp} = \omega_L + \delta\omega_{\perp}, \quad (1)$$

where the frequency shift $\delta\omega_{\perp}$ from the Larmor value $\omega_L = gH$ is due to the action of the dipole-dipole (spin-orbit) forces described by the potential

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$$U_D = -\frac{1}{2}\chi\left(\frac{\Omega}{g}\right)^2(\hat{d} \cdot \hat{l})^2, \quad (2)$$

where \hat{d} is the axis of magnetic anisotropy of $^3\text{He-A}$ order parameter. In a strong magnetic field ($\omega_L \gg \Omega$) the transverse NMR frequency shift

$$\delta\omega_{\perp} = -\frac{1}{S} \frac{\partial \bar{U}_D}{\partial \cos \beta}, \quad (3)$$

where \bar{U}_D denotes the time -averaged spin-orbit potential, $S = |\mathbf{S}| = \chi H/g$ is an equilibrium magnitude of spin density and β marks the tipping angle of $\mathbf{S}(t)$ precessing about the direction of an applied magnetic field $\mathbf{H} = H\hat{z}$ ($S_z = S \cos \beta$).

In above-mentioned experiments (Refs.[1,2,3]) the magnetic field was directed along deformation axis of aerogel samples filled with superfluid A-like phase and the information about spin dynamics could be extracted from $\delta\omega_{\perp}(\beta)$. In what follows we consider the case where the orientation of \mathbf{H} can be inclined by an angle ϑ with respect to the axis of deformation of aerogel. In this situation the observed frequency shift $\delta\omega_{\perp}(\beta, \vartheta)$ can be explored in (β, ϑ) plane allowing to cover entire stability region of coherently spin-precessing modes. The experiments using the rotation of the magnetic field with respect to aerogel deformation axis was, in particular, undertaken in Ref.[4]. Our theoretical consideration contains some suggestions for further experimental efforts.

In order to describe the configuration where the external magnetic field $\mathbf{H} = H\hat{z}$ is inclined with respect to the axis of a global deformation of aerogel we consider two coordinate frames: an orbital frame $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$ "attached" to the cylindrical experimental cell (with $\hat{\zeta}$ oriented along the cell axis) and the spin space frame $(\hat{x}, \hat{y}, \hat{z})$. The orbital anisotropy axis is given as:

$$\hat{l} = (\hat{\xi} \cos \phi_l + \hat{\eta} \sin \phi_l) \sin \lambda + \hat{\zeta} \cos \lambda. \quad (4)$$

In the spin-precessing regime the magnetic anisotropy axis $\hat{d}(t)$ is rotating about an instantaneous orientation of $\mathbf{S}(t)$, so that

$$\hat{d}(t) = \mathbf{R}(\alpha, \beta, \gamma) \hat{x} \perp \mathbf{S}(t) = S \mathbf{R}(\alpha, \beta, \gamma) \hat{z}, \quad (5)$$

where (α, β, γ) are Euler angles describing 3D rotations in the spin space. In order to construct $\delta\omega_{\perp}(\beta, \vartheta)$ we have to take into account that for the orientation of \mathbf{H} confined in the (ξ, ζ) plane

$$\begin{aligned} \hat{d} \cdot \hat{l} = & d_x (\sin \lambda \cos \phi_l \cos \vartheta - \cos \lambda \sin \vartheta) + \\ & + d_y (\sin \lambda \sin \phi_l) + \\ & + d_z (\sin \lambda \cos \phi_l \sin \vartheta + \cos \lambda \cos \vartheta). \end{aligned} \quad (6)$$

In what follows we concentrate on two special orbital configurations realized experimentally:

- a) axially compressed aerogel with $\lambda = 0$;

b) axially stretched aerogel with $\lambda = \pi/2$.

The results of our investigations are presented in the following sequence. In Sec.2 the spin-precessing mode of the A-like phase in axially compressed aerogel environment is considered. In Sec.3 the spin-precessing mode of the A-like phase in axially stretched aerogel environment is explored using the Volovik U(1)LIM model. The conclusions are presented in Sec.4.

2 Spin-precessing Mode of A-like Phase in Axially Compressed Aerogel.

In the considered long-ranged orbital configuration with $\lambda = 0$ the orbital phase ϕ_l is an irrelevant variable and according to Eq.(6) the spin-orbit function

$$\begin{aligned} f &= (\hat{d} \cdot \hat{l})^2 = (d_x \sin \vartheta - d_z \cos \vartheta)^2 = \\ &= d_z^2 + (d_x^2 - d_z^2) \sin^2 \vartheta - d_x d_z \sin 2\vartheta. \end{aligned} \quad (7)$$

In an explicit form

$$\begin{aligned} f(t) &= \frac{1}{4} \left\{ 2 \sin^2 \beta (1 + \cos 2\gamma) + \left[-1 + 3 \cos^2 \beta + \right. \right. \\ &\quad + \frac{1}{2} (1 + \cos \beta)^2 \cos 2(\alpha + \gamma) + \frac{1}{2} (1 - \cos \beta)^2 \cos 2(\alpha - \gamma) - \\ &\quad \left. \left. - \sin^2 \beta (\cos 2\alpha + 3 \cos 2\gamma) \right] \sin^2 \vartheta + \right. \\ &\quad + \sin \beta [2 \cos \beta \cos \alpha + (1 + \cos \beta) \cos(\alpha + 2\gamma) - \\ &\quad \left. - (1 - \cos \beta) \cos(\alpha - 2\gamma)] \sin 2\vartheta \right\}, \end{aligned} \quad (8)$$

and at $S = S_{eq} = (\chi/g^2)\omega_L$ the phase $\phi = \alpha + \gamma$ is a slow variable. As a result, $f(t)$ can be decomposed in standard way [5] as

$$f(t) = \bar{f} + \tilde{f}(t), \quad (9)$$

where the time-averaged (Van der Pol) part

$$\bar{f} = \frac{1}{2} \sin^2 \beta + \frac{1}{4} \left[-1 + 3 \cos^2 \beta + \frac{1}{2} (1 + \cos \beta)^2 \cos 2\phi \right] \sin^2 \vartheta, \quad (10)$$

while the rapidly time-fluctuating contribution

$$\begin{aligned} \tilde{f}(t) = \frac{1}{4} \Bigg\{ & 2 \sin^2 \beta \cos 2\gamma + \left[\frac{1}{2} (1 - \cos \beta)^2 \cos 2(\alpha - \gamma) - \right. \\ & \left. - \sin^2 \beta (\cos 2\alpha + 3 \cos 2\gamma) \right] \sin^2 \vartheta + \left[\sin 2\beta \cos \alpha + \right. \\ & \left. + \sin \beta (1 + \cos \beta) \cos(\alpha + 2\gamma) - \right. \\ & \left. - \sin \beta (1 - \cos \beta) \cos(\alpha - 2\gamma) \right] \sin 2\vartheta \Bigg\}, \end{aligned} \quad (11)$$

gives life to the small amplitude high-frequency spin fluctuations superimposed on the coherently spin-precessing modes emerging from the Van der Pol picture. As follows from Eq.(10), in orbital state with $\lambda = 0$

$$\bar{U}_D = -\frac{1}{2} \chi \left(\frac{\Omega}{g} \right)^2 \left[\frac{1}{2} \sin^2 \beta + \frac{1}{4} \left(-1 + 3 \cos^2 \beta + \frac{1}{2} (1 + \cos \beta)^2 \cos 2\phi \right) \sin^2 \vartheta \right]. \quad (12)$$

The phase ϕ entering in Eq.(12) can be fixed by minimizing \bar{U}_D at $\phi = \phi_{st} = (0, \pi)$:

$$\bar{U}_D = -\frac{1}{2} \chi \left(\frac{\Omega}{g} \right)^2 \left[\frac{1}{2} \sin^2 \beta + \frac{1}{8} (-1 + 2 \cos \beta + 7 \cos^2 \beta) \sin^2 \vartheta \right], \quad (13)$$

and according to Eq.(3) the dipole frequency shift

$$\delta \omega_{\perp}(\beta, \vartheta) = \left(\frac{\Omega^2}{2\omega_L} \right) \left[-\cos \beta + \frac{1}{4} (1 + 7 \cos \beta) \sin^2 \vartheta \right]. \quad (14)$$

The stability criterion (the concavity of \bar{U}_D with respect to $\cos \beta$) is

$$\frac{\partial^2 \bar{U}_D}{\partial (\cos \beta)^2} > 0 \Rightarrow \cos^2 \vartheta > \frac{3}{7}, \quad (15)$$

which is in accordance with the conclusion found in Ref.[6].

In order to reach a contact with the description of the coherently spin-precessing state in terms of the magnon BEC [7] one has to take into account that an excited spin-dynamical mode with fixed frequency and phase is characterized by the Bose quasiparticles - magnons with density

$$n_M = |\psi|^2 = S - S_z = S(1 - \cos \beta), \quad (16)$$

so that Eq.(13) for \bar{U}_D can be transcribed as

$$\bar{U}_D(\beta, \vartheta) = -\frac{1}{2} \chi \left(\frac{\Omega}{g} \right)^2 \left[\sin^2 \vartheta + \frac{|\psi|^2}{S} \cos 2\vartheta - \frac{7}{8} (\cos^2 \vartheta - \cos^2 \vartheta_o) \frac{|\psi|^4}{S^2} \right], \quad (17)$$

where $\cos^2 \vartheta_o = 3/7$. From Eq.(17) it follows that magnon-magnon interaction potential

$$U_M(\vartheta) = \frac{7}{4^2} \chi \left(\frac{\Omega}{g} \right)^2 (\cos^2 \vartheta - \cos^2 \vartheta_o), \quad (18)$$

and the magnon-magnon repulsion regime ($\cos^2 \vartheta > \cos^2 \vartheta_o$) reproduces the stability criterion (15) of coherent spin-precessing mode, as expected.

Now, the dipole frequency shift for an axially compressed aerogel (at $\lambda = 0$) can be represented as

$$\begin{aligned} \delta\omega_{\perp}(\beta, \vartheta) &= \left(\frac{\Omega^2}{2\omega_L} \right) \left[-\cos 2\vartheta + \frac{7}{4} (\cos^2 \vartheta - \cos^2 \vartheta_o) (1 - \cos \beta) \right] = \\ &= \frac{1}{4} \left(\frac{\Omega^2}{2\omega_L} \right) [\sin^2 \vartheta - 7(\cos^2 \vartheta - \cos^2 \vartheta_o) \cos \beta]. \end{aligned} \quad (19)$$

In Ref.[4] it was shown, in particular, that in linear cw NMR regime ($\beta \rightarrow 0$) the experimental data are well described by the formula

$$\delta\omega_{\perp}(0, \vartheta) = - \left(\frac{\Omega^2}{2\omega_L} \right) \cos 2\vartheta. \quad (20)$$

On the other hand, in the pulsed NMR case for $\vartheta = \pi/2$ it was found that the dipole frequency shift is described by the Brinkman-Smith formula

$$\delta\omega_{\perp}(\beta, \pi/2) = \left(\frac{\Omega^2}{2\omega_L} \right) \frac{1}{4} (1 + 3 \cos \beta). \quad (21)$$

It should be remarked that according to Ref.[2] the measured β -dependence of dipole frequency shift at $\vartheta = 0$

$$\delta\omega_{\perp}(\beta, 0) = - \left(\frac{\Omega^2}{2\omega_L} \right) \cos \beta. \quad (22)$$

All of the mentioned experimental results are in accordance with general Eq.(19). At the same time, the answer for an axially compressed aerogel case (with $\lambda = 0$) contains some hints to stimulate further experimental efforts. In addressing Eq.(19) we see that at the edge of the stability of coherently spin-precessing mode realized at the magnetic field tilting angle $\vartheta = \vartheta_o$ the dipole frequency shift should be independent of spin-tipping angle β :

$$\delta\omega_{\perp}(\beta, \vartheta_o) \equiv \frac{1}{7} \left(\frac{\Omega^2}{2\omega_L} \right). \quad (23)$$

On the other hand, at $\cos \beta_o = -1/7$ the dipole frequency shift should be independent of the magnetic field inclination angle ϑ :

$$\delta\omega_{\perp}(\beta_o, \vartheta) \equiv \frac{1}{7} \left(\frac{\Omega^2}{2\omega_L} \right). \quad (24)$$

This behaviour of $\delta\omega_{\perp}(\beta, \vartheta)$, given by Eq.(19), is clearly seen in Fig.1.

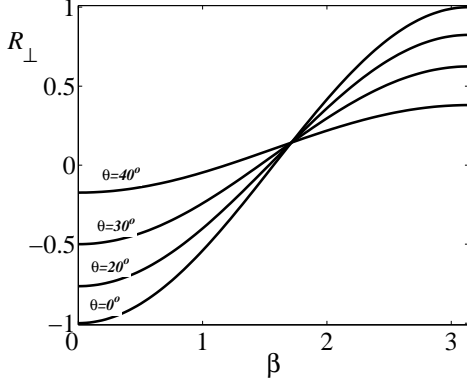


Fig. 1 Dependence of normalized dipole frequency shift $R_{\perp}(\beta, \vartheta) = \delta\omega_{\perp}(\beta, \vartheta)/(\Omega^2/2\omega_L)$ on β at various inclination angles ϑ .

It is to be mentioned that Eq.(19) describing the dipole frequency shift $\delta\omega_{\perp}(\beta, \vartheta)$ is applicable to $^3\text{He-A}$ confined in the narrow gap with orbital axis \hat{l} pinned to the normal of parallel plates. The corresponding experiments [8] were performed in the case of $\mathbf{H} \parallel \hat{l}$ ($\vartheta = 0$). Unfortunately no attempt to explore the ϑ -dependence of $\delta\omega_{\perp}(\beta, \vartheta)$ was undertaken at that time.

3 Spin-precessing Mode of A-like Phase in Axially Stretched Aerogel.

Now, we proceed to an axially stretched (radially squeezed) aerogel case corresponding to the orbital configuration with $\lambda = \pi/2$. In referring to Eq.(6) it is readily seen that in this situation

$$(\hat{dl})^2 = d_x^2 \cos^2 \phi_l \cos^2 \vartheta + d_y^2 \sin^2 \phi_l + d_z^2 \cos^2 \phi_l \sin^2 \vartheta + d_x d_y \sin 2\phi_l \cos \vartheta + d_x d_z \cos^2 \phi_l \sin 2\vartheta + d_y d_z \sin 2\phi_l \sin \vartheta. \quad (25)$$

According to the $U(1)$ LIM model [3] for the fully randomized ϕ_l

$$\langle \sin^2 \phi_l \rangle = \langle \cos^2 \phi_l \rangle = \frac{1}{2}, \quad \langle \sin 2\phi_l \rangle = 0, \quad (26)$$

and it is concluded that in an axially stretched aerogel

$$\begin{aligned} (\hat{dl})^2 &= \frac{1}{2}(d_x^2 \cos^2 \vartheta + d_y^2 + d_z^2 \sin^2 \vartheta + d_x d_z \sin 2\vartheta) = \\ &= \frac{1}{2}(\hat{d}^2(t) - (d_x \sin \vartheta - d_z \cos \vartheta)^2) = -\frac{1}{2}f(t) + \text{const}, \end{aligned} \quad (27)$$

where $f(t)$ is given by Eq.(8). In constructing the time-averaged (Van der Pol) dipole-dipole potential

$$\langle \bar{U}_D \rangle = \frac{1}{4} \chi \left(\frac{\Omega}{g} \right)^2 \bar{f}, \quad (28)$$

we see that according to Eq.(10) it is minimized at $\phi = \pi/2$ and for the dipole frequency shift $\delta\omega_\perp$ the following answer is obtained

$$\delta\omega_\perp(\beta, \vartheta) = \frac{1}{2} \left(\frac{\Omega^2}{2\omega_L} \right) \left[\cos\beta + \frac{1}{4}(1 - 5\cos\beta) \sin^2\vartheta \right]. \quad (29)$$

In particular,

$$\delta\omega_\perp = \left(\frac{\Omega^2}{2\omega_L} \right) \begin{cases} \frac{1}{2} \cos\beta, & \vartheta = 0 \\ \frac{1}{8}(1 - \cos\beta), & \vartheta = \pi/2. \end{cases} \quad (30)$$

In the magnon BEC representation

$$\langle \bar{U}_D \rangle = \frac{1}{4} \chi \left(\frac{\Omega}{g} \right)^2 \left[\frac{|\psi|^2}{S} \cos^2\vartheta - \frac{5}{8} \frac{|\psi|^4}{S^2} (\cos^2\vartheta - \cos^2\vartheta_o) \right], \quad (31)$$

where $\cos^2\vartheta_o = 1/5$. Consequently, the magnon-magnon interaction potential

$$U_M(\vartheta) = -\frac{5}{2^5} \chi \left(\frac{\Omega}{g} \right)^2 (\cos^2\vartheta - \cos^2\vartheta_o), \quad (32)$$

and it is repulsive at $\cos^2\vartheta < \cos^2\vartheta_o$. The stability region of spin-precessing mode in the case of an axially stretched aerogel environment ($\lambda = \pi/2$), due to the $U(1)$ LIM effect, is confident in a narrow gap

$$0 \leq \cos^2\vartheta < \frac{1}{5}, \quad (33)$$

being essentially different from axially compressed aerogel case with $\lambda = 0$ (see Eq.(15) and Fig.2). This means, in particular, that the spin-precessing mode realized at $\vartheta = 0$ (see Eq.(30)) and discovered experimentally [3] is an unstable state. It would be desirable to support this conclusion by observation of the fast decay of FIS appropriate to an unstable spin-precessing mode. In an open geometry such test of the instability of spin-precession in $^3\text{He-A}$ for the Leggett orbital configuration have been performed in Refs.[9, 10] confirming the prediction made in Ref.[11]. On the other hand, the expectation for the transverse orientation of $\mathbf{H}(\vartheta = \pi/2)$, see Eq.(30), which is in the limits of stability region (33), is still to be confirmed experimentally.

Returning back to the β -representation we find that in an axially stretched aerogel case

$$\begin{aligned} \delta\omega_\perp(\beta, \vartheta) &= \frac{1}{2} \left(\frac{\Omega^2}{2\omega_L} \right) \left[\cos^2\vartheta - \frac{5}{4} (\cos^2\vartheta - \cos^2\vartheta_o)(1 - \cos\beta) \right] = \\ &= \frac{1}{8} \left(\frac{\Omega^2}{2\omega_L} \right) [\sin^2\vartheta + 5(\cos^2\vartheta - \cos^2\vartheta_o) \cos\beta]. \end{aligned} \quad (34)$$

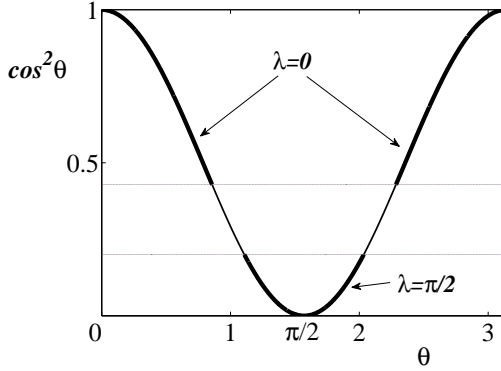


Fig. 2 Stability regions of coherently spin-precessing modes in axially compressed ($\lambda = 0$) and axially stretched ($\lambda = \pi/2$) aerogel.

4 Conclusion

In this article the specific features of the NMR spectra of the A-like phase of superfluid ^3He in uni-axially deformed aerogel environment is considered in presence of an inclined magnetic field with respect to global deformation axis. Two special orbital configurations, relevant to experimentally investigated cases, are analyzed: a) an axially compressed aerogel with orbital anisotropy axis \hat{l} pinned to deformation axis; b) an axially stretched aerogel with \hat{l} confined in transverse plane with respect to deformation axis. The later case is treated in terms of $U(1)$ LIM model. In the mentioned cases the frequency shift $\delta\omega(\beta, \vartheta)$ from the Larmor value $\omega_L = gH$ is constructed. Among other things, the stability limits of coherently spin-precessing modes with respect to magnetic field inclination angle ϑ are found. In particular, it is shown that in an axially stretched aerogel environment the $U(1)$ LIM effect reduces the stability region to a narrow gap around $\vartheta = \pi/2$. Some suggestions for further experimental efforts are given.

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